

# Deterministic Letter-to-Sound Transduction in the Taxi/Grimm Corpus Indexing System

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# The Big Picture

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## The Big Picture

- ▶ Grimm Quotation Evidence Corpus
- ▶ Taxi Corpus Indexing System

## Letter-to-Sound Conversion

- ▶ Festival LTS Rulesets
- ▶ Failed Attempts
  - ... or: “*Why lextools Won’t Cut The Butter*”
- ▶ LTS Transducer Generation

## Results

- ▶ Morphological Coverage

## Demonstration

- ▶ Taxi/Grimm HTTP Query Server



# Grimm Corpus

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## Sources

- ▶ *Deutsches Wörterbuch* originally by Jacob and Wilhelm Grimm
- ▶ SGML sources provided by the Universität Trier
- ▶ Conversion to XML at the BBAW

## Quotation Evidence Corpus

- ▶ Verse quotations encoded in sources
  - 382,766 verse quotations
  - 6,581,509 tokens
  - 557,271 distinct orthographic types
- ▶ Prose quotation extraction work in progress



# Grimm Corpus: Problems

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## Historical Orthography

- ▶ Variant orthographical conventions
- ▶ Disparate sources
  - (incl. Walther von der Vogelweide, ca. A.D. 1200)
- ▶ Not supported by conventional analysis tools
  - (TAGH, moot, etc.)

## Examples

- ▶ “fröhlich” ↪ *frölich*, *fröhlich*, *vroëlich*, *frælich*, *frölich*,  
*fröhlich*, *vrölich*, *fröhlig*, *frölig*, . . .
- ▶ “herzenleid” ↪ *hertzenleid*, *herzenleid*, *herzenleit*,  
*hertzenleyd*, *hertzenleidt*, *herzenlaid*, *hertzenlaid*,  
*hertzenlaidt*, *hertzenlaydt*, *herzenleyd*, . . .



# Historical Orthography Workaround

## Intuitions

- ▶ Pronunciation endures longer than orthography
- ▶ Phonetic form is a better indicator of lexical identity than orthographic form

## Workaround Idea

- ▶ Map each orthographic type to a phonetic form
  - ▶ Letter-to-Sound (LTS) Translation Module
  - ▶ ideally, as a finite-state transducer (FST)
- ▶ Adapt synchronically oriented tools to operate on phonetic forms
  - ▶ First step: extend known analyses to phonetically identical types: extensional join



# Taxi Indexing System

## TAXI: Another XML Index

- ▶ Abstract index for XML-structured text corpora
- ▶ MySQL relational database for backend storage

## Build Subsystem

- ▶ Converts raw SGML Sources to TAXI XML
- ▶ Entity resolution, tokenization, document analysis

## Indexing Subsystem

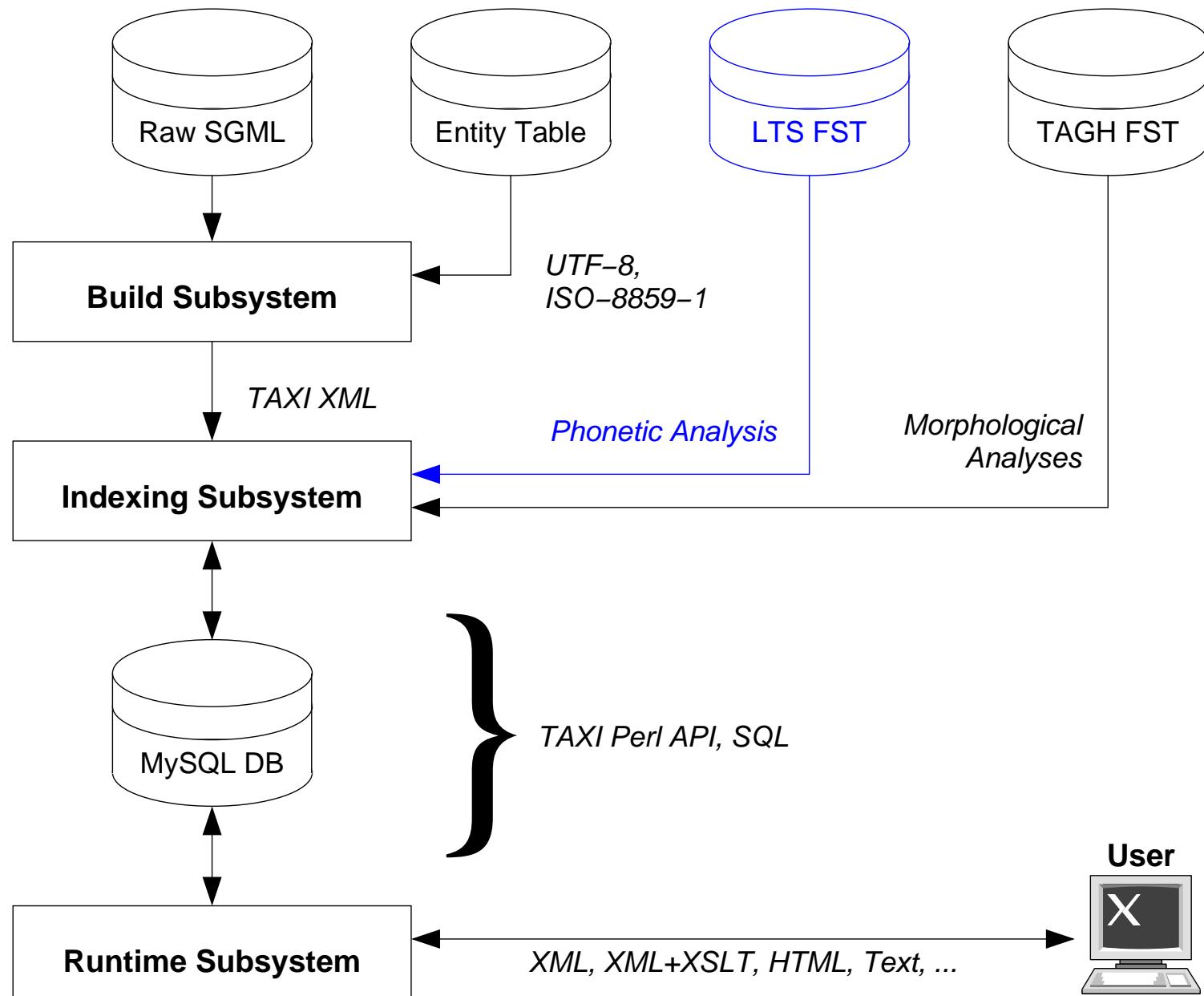
- ▶ Backend MySQL database administration
- ▶ Performs phonetic & morphological analysis

## Runtime Subsystem

- ▶ User query handling (parsing, retrieval, formatting)
- ▶ HTTP server mode for remote queries



# Taxi/Grimm System Architecture



# Letter-to-Sound Conversion

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## The Big Picture

- ▶ Grimm Quotation Evidence Corpus
- ▶ Taxi Corpus Indexing System

## Letter-to-Sound Conversion

- ▶ Festival LTS Rulesets
- ▶ Failed Attempts
  - ... or: “*Why lextools Won’t Cut The Butter*”
- ▶ LTS Transducer Generation

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# LTS Conversion: Overview

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## Festival Letter-to-Sound Rulesets

- ▶ Festival Text-to-Speech System
- ▶ IMS German Festival Package
- ▶ Syntax & Semantics of Festival LTS Rules

## Failed Attempts

- ▶ AT&T Tools: lexcomplex, lexrulecomp
- ▶ Roche & Schabes' Brill Tagger Emulation

## LTS Transducer Generation

- ▶ Aho-Corasick Pattern Matchers
- ▶ Intermediate Alphabets
- ▶ Output Filter



# Festival & IMS German Festival

## Festival TTS System

(*Black & Taylor, 1997*)

<http://www.cstr.ed.ac.uk/projects/festival>

- ▶ Free modular **Text-to-Speech System** (X11 license)
- ▶ Implemented in **SCHEME** and **C++**
- ▶ **Abstract**: language- and model-independent
- ▶ **Extensible**: research- & development-oriented
- ▶ **SLOW**

## IMS German Festival Module

(*Möhler et al., 2001*)

<http://www.ims.uni-stuttgart.de/phonetik/synthesis>

- ▶ German language support **module** for festival
- ▶ Available free of cost for **research purposes**
- ▶ Unmaintained: **broken** in festival versions  $\geq 1.4.1$
- ▶ Includes a working **German LTS rule-set**



# Festival LTS Rules: Syntax

## Alphabets

- ▶  $\Sigma_g$ : finite input alphabet (**graphemes**)
- ▶  $\Sigma_p$ : finite output alphabet (**phones**)
- ▶  $\Sigma_g \cap \Sigma_p = \emptyset$

## Rules

- ▶  $Rule ::= (\alpha[\beta]\gamma \rightarrow \pi) \quad \in \Sigma_g^* \times \Sigma_g^+ \times \Sigma_g^* \times \Sigma_p^*$
- ▶ Rule target length:  $|(\alpha[\beta]\gamma \rightarrow \pi)| = |\beta|$

## Ruleset

- ▶ Strictly ordered finite sequence
- ▶  $R = \langle r_1, \dots, r_{n_R} \rangle$
- ▶  $r_i \prec r_j$  iff  $i < j$  for  $1 \leq i, j \leq n_R$



# Festival LTS Rules: Semantics 1

## Informal Procedural Semantics

- ▶ Input is processed from **left to right**
- ▶ The **first matching rule** is applied at each position

## Rule Matching

[Notation:  $r \sim_i w$ ]

- ▶ The rule  $r = (\alpha[\beta]\gamma \rightarrow \pi)$  **matches** an input string  $w = w_1 \cdots w_n$  at position  $i$  iff:

$$w_{i-|\alpha|} \cdots w_{i+|\beta\gamma|} = \alpha\beta\gamma$$

## Rule Applicability

[Notation:  $r \approx_i w$ ]

- ▶ The rule  $r$  is **applicable** to  $w$  at  $i$  iff:

$$r \sim_i w \quad \text{and} \quad \forall r' \in R. r' \prec r \implies r' \not\sim_i w$$



# Festival LTS Rules: Semantics 2

## Configurations

- $\langle \underbrace{w}_{\text{input}}, \underbrace{i}_{\text{position}}, \underbrace{p}_{\text{output}} \rangle \in \Sigma_g^* \times \mathbb{N} \times \Sigma_p^*$

## Successive Rule Application

- $\underbrace{\langle w, i, p \rangle}_{\text{source config}} \vdash_R \underbrace{\langle w, i + |\beta|, p\pi \rangle}_{\text{sink config}}$  iff  $\underbrace{(\alpha[\beta]\gamma \rightarrow \pi) \approx_i w}_{\text{use first applicable rule}}$

## LTS Function

- $LTS : \Sigma_g^* \rightarrow \Sigma_p^* : w \mapsto p$  iff:  
$$\underbrace{\langle w, 1, \varepsilon \rangle}_{\text{initial configuration}} \vdash_R^* \underbrace{\langle w, |w| + 1, p \rangle}_{\text{final configuration}}$$



# Festival LTS Rules: Algorithm

**function** LTS

**Parameter:** ordered rules  $\langle R, \prec \rangle$

**Paramter:** word  $w \in \Sigma_g^*$

**Returns:** phonetic string  $p \in \Sigma_p^*$

**begin**

$i := 1$

*/\* Initialize: read position \*/*

$p := \varepsilon$

*/\* Initialize: output buffer \*/*

**while** ( $i < |w|$ ) **do**

**if** ( $R \sim_i w = \emptyset$ ) **then error;**   */\* No match found \*/*

$(\alpha[\beta]\gamma \rightarrow \pi) = \min_{\prec} R \sim_i w$            */\* First match \*/*

$p := p\pi$                                            */\* Accumulate output \*/*

$i := i + |\beta|$                                    */\* Consume input \*/*

**end while**

**return**  $p$

**end function**



# Festival LTS Rules: Example

## Rules

[ a ] \$C \$C → a

[ a ] → a:

\$Vb [c h] → x

[ c ] → k

[ e ] → Θ

# [ s ] \$V → z

[ s ] → s

## Example Ruleset (with symbol classes)



# Festival LTS Rules: Example

| Rules             | Input |
|-------------------|-------|
| [ a ] \$C \$C → a | sache |
| [ a ] → a:        |       |
| \$Vb [ c h ] → x  |       |
| [ c ] → k         |       |
| [ e ] → Θ         |       |
| # [ s ] \$V → z   |       |
| [ s ] → s         |       |

## Example input word



# Festival LTS Rules: Example

| Rules             | Input   |
|-------------------|---------|
| [ a ] \$C \$C → a | #sache# |
| [ a ] → a:        |         |
| \$Vb [ c h ] → x  |         |
| [ c ] → k         |         |
| [ e ] → Θ         |         |
| # [ s ] \$V → z   |         |
| [ s ] → s         |         |

## BOS & EOS markers



# Festival LTS Rules: Example

| Rules             | Input     |
|-------------------|-----------|
| [ a ] \$C \$C → a | # [sache# |
| [ a ] → a:        |           |
| \$Vb [c h] → x    |           |
| [ c ] → k         |           |
| [ e ] → Θ         |           |
| # [ s ] \$V → z   |           |
| [ s ] → s         |           |

Initialize read position



# Festival LTS Rules: Example

| Rules             | Input    | : | Output |
|-------------------|----------|---|--------|
| [ a ] \$C \$C → a | #[sache# | : | ε      |
| [ a ] → a:        |          |   |        |
| \$Vb [c h] → x    |          |   |        |
| [ c ] → k         |          |   |        |
| [ e ] → Θ         |          |   |        |
| # [ s ] \$V → z   |          |   |        |
| [ s ] → s         |          |   |        |

Initialize output buffer



# Festival LTS Rules: Example

| Rules         |                      | Input    | : | Output     |
|---------------|----------------------|----------|---|------------|
| [ a ] \$C \$C | $\rightarrow a$      | #[sache# | : | $\epsilon$ |
| [ a ]         | $\rightarrow a:$     | #[sache# | : | $\epsilon$ |
| \$Vb [c h]    | $\rightarrow x$      |          |   |            |
| [ c ]         | $\rightarrow k$      |          |   |            |
| [ e ]         | $\rightarrow \theta$ |          |   |            |
| # [ s ] \$V   | $\rightarrow z$      |          |   |            |
| [ s ]         | $\rightarrow s$      |          |   |            |

Find matching rules



# Festival LTS Rules: Example

| Rules             |  | Input                     | : | Output        |
|-------------------|--|---------------------------|---|---------------|
| [ a ] \$C \$C → a |  | #[sache#                  | : | $\varepsilon$ |
| [ a ] → a:        |  | <u>#</u> <u>[s]</u> ache# | : | $z$           |
| \$Vb [ch] → x     |  |                           |   |               |
| [ c ] → k         |  |                           |   |               |
| [ e ] → Θ         |  |                           |   |               |
| # [ s ] \$V → z   |  |                           |   |               |
| [ s ] → s         |  |                           |   |               |

Apply first matching rule



# Festival LTS Rules: Example

| Rules             |  | Input              | : | Output |
|-------------------|--|--------------------|---|--------|
| [ a ] \$C \$C → a |  | #[sache#           | : | ε      |
| [ a ] → a:        |  | #[s]ache#          | : | z      |
| \$Vb [ch] → x     |  | #s <a> [a]che#</a> | : | za     |
| [ c ] → k         |  |                    |   |        |
| [ e ] → Θ         |  |                    |   |        |
| # [ s ] \$V → z   |  |                    |   |        |
| [ s ] → s         |  |                    |   |        |

Increment, match & apply



# Festival LTS Rules: Example

| Rules             |  | Input      | : | Output        |
|-------------------|--|------------|---|---------------|
| [ a ] \$C \$C → a |  | #[sache#   | : | $\varepsilon$ |
| [ a ] → a:        |  | #[_s]ache# | : | z             |
| \$Vb [c h] → x    |  | #s[a]che#  | : | za            |
| [ c ] → k         |  | #sa[_ch]e# | : | zax           |
| [ e ] → Θ         |  |            |   |               |
| # [ s ] \$V → z   |  |            |   |               |
| [ s ] → s         |  |            |   |               |

Increment, match & apply



# Festival LTS Rules: Example

| Rules             |  | Input      | : | Output        |
|-------------------|--|------------|---|---------------|
| [ a ] \$C \$C → a |  | #[sache#   | : | $\varepsilon$ |
| [ a ] → a:        |  | #[_s]ache# | : | z             |
| \$Vb [ch] → x     |  | #s[a]che#  | : | za            |
| [ c ] → k         |  | #sa[_ch]e# | : | zax           |
| [ e ] → $\Theta$  |  | #sach[e]#  | : | zax $\Theta$  |
| # [ s ] \$V → z   |  |            |   |               |
| [ s ] → s         |  |            |   |               |

Increment, match & apply



# Festival LTS Rules: Example

| Rules             |  | Input     | : | Output        |
|-------------------|--|-----------|---|---------------|
| [ a ] \$C \$C → a |  | #[sache#  | : | $\varepsilon$ |
| [ a ] → a:        |  | #[s]ache# | : | z             |
| \$Vb [ch] → x     |  | #s[a]che# | : | za            |
| [ c ] → k         |  | #sa[ch]e# | : | zax           |
| [ e ] → ə         |  | #sach[e]# | : | zaxə          |
| # [ s ] \$V → z   |  | sache     | : | /zaxə/        |
| [ s ] → s         |  |           |   |               |

Final output



# Festival LTS Rules: Analysis

## Desideratum

- ▶ FST implementation of  $LTS(\cdot)$  function

## Clearly...

- ▶ Festival LTS rule application is deterministic
- ▶ Each rule  $(\alpha[\beta]\gamma \rightarrow \pi)$  defines a regular transduction  $\alpha(\beta : \pi)\gamma$

## Problems

- ▶ Rule precedence (bleeding)
- ▶ Rule-dependent position incrementation  
(derivability  $\not\equiv$  WFST best-path)
- ▶ Rule context lookahead / -behind



# Failed Attempt: lexcomplex

## Idea

- ▶ Convert each LTS rule  $r$  to a regular expression  $\llbracket r \rrbracket$ :
  - ▶  $\llbracket (\alpha[\beta]\gamma \rightarrow \pi) \rrbracket = \alpha(\beta : \pi)\gamma$
- ▶ Compile results with lexcomplex:
  - ▶  $\llbracket R \rrbracket_{lex} = \bigcup_{r \in R} \llbracket r \rrbracket$

## Problems

- ▶ No rule precedence  $\implies$  non-deterministic FST
  - ▶ Workaround: use costs to simulate precedence
- ▶ Only one rule is applied by  $\llbracket R \rrbracket$ 
  - ▶ Kleene-closure  $\llbracket R \rrbracket_{lex}^*$  also consumes contexts
  - ▶ No lookahead / -behind

OUT-OF-CHEESE ERROR: REDO FROM START



# Failed Attempt: lexcomplex -m

## Idea

- ▶ Convert each  $r \in R$  to regex  $\llbracket r \rrbracket$  as above
- ▶ Compile results with lexcomplex -m:
  - ▶  $\llbracket R \rrbracket_{lex-m} = ((Id(\Sigma) - Prj_1(\llbracket R \rrbracket_{lex})) \cup \llbracket R \rrbracket_{lex})^*$

## Problems

- ▶ -m mode designed for single symbol input
- ▶ No rule precedence, non-deterministic FST
- ▶  $\llbracket R \rrbracket_{lex-m}$  also consumes contexts
  - ▶ No lookahead / -behind

OUT-OF-CHEESE ERROR: REDO FROM START



# Failed Attempt: lexcomplex -M

## Idea

- ▶ Convert each  $r \in R$  to regex  $\llbracket r \rrbracket$  as above
- ▶ Compile results with lexcomplex -M:
  - ▶  $\llbracket R \rrbracket_{lex-M} = \text{undocumented}$

## Problems

- ▶ -M mode designed for parallel batch rewrites
- ▶ No rule precedence, non-deterministic FST
- ▶  $\llbracket R \rrbracket_{lex-M}$  consumes contexts
  - ▶ No lookahead / -behind

OUT-OF-CHEESE ERROR: REDO FROM START



# Failed Attempt: lexrulecomp

## Idea

- ▶ Convert each  $r \in R$  to a rewrite rule  $\llbracket r \rrbracket_{lexrul}$ :
  - ▶  $\llbracket (\alpha[\beta]\gamma \rightarrow \pi) \rrbracket_{lexrul} = \beta \rightarrow \pi / \alpha - \gamma$
- ▶ Compile result with lexrulecomp:
  - ▶  $\llbracket R \rrbracket_{lexrul} = \text{undocumented}$
- ▶ Left-to-right obligatory mode

## Problems

- ▶  $\llbracket R \rrbracket_{lexrul}$  handles overlapping contexts improperly
  - ▶ Bad lookahead / -behind ?
  - ▶ Bad position incrementation ?

OUT-OF-CHEESE ERROR: REDO FROM START (AGAIN)



# LTS FST: Method

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## Step 1: Rule Match Detection

- ▶ Aho-Corasick Pattern Matchers (ACPMs)
- ▶ Distinguish **left** vs. **right** of current I/O position
- ▶ Intermediate **alphabets**: matched rule subsets
- ▶ Composition of (reversed) partial ACPMs models  
LTS **configurations**

## Step 2: Output Filter

- ▶ Composed with match-detection FST
- ▶ Selects **applicable** rules
- ▶ Simulates **position increment**



# Aho-Corasick Pattern Matching

## Given

- ▶ A set  $P = \{p_1, \dots, p_{n_P}\} \subseteq \Sigma^*$  of patterns

## Task

- ▶ Identify all occurrences of any pattern  $p \in P$  in an input string  $w \in \Sigma^*$

## Aho-Corasick Algorithm (Aho & Corasick, 1975)

- ▶ Constructs a pattern-matching FST  
 $\mathcal{AC}_P : \Sigma^* \rightarrow \mathfrak{P}(P)^*$
  - ▶ Linear runtime complexity:  $\mathcal{O} = \mathcal{O}(|w|)$ 
    - ▶ Really  $\mathcal{O}(|w| + \sum_{i=1}^{n_P} |p_i| + \sum_{j=1}^{|w|} |P \sim_j w|)$
- ... but who's counting?*



# ACPM Construction Sketch

## Input

- ▶ Prefix Tree Acceptor (trie) for  $P$

## Output

- ▶ Finite-state transducer  $\mathcal{AC}_P : \Sigma^* \rightarrow \mathfrak{P}(P)^*$  such that
$$\mathcal{AC}_P(w) = \bigodot_{i=1}^{|w|} \{p \in P \mid p = w_{i-|p|}..w_i\}$$

## Method

- ▶ Transition function
- ▶ Failure function
- ▶ Output function

`goto` :  $Q \times \Sigma \rightarrow Q$

`fail` :  $Q \rightarrow Q$

`out` :  $Q \rightarrow \mathfrak{P}(P)$

Together, `goto`, `fail`, and `out` define a conventional arc-dependent input-deterministic transition function

$\delta : Q \times \Sigma \rightarrow Q \times \mathfrak{P}(P)$



# ACPM Limitations & Workarounds

## Delayed Output

- ▶ ACPM output occurs at **pattern terminus**
- ▶ **Problem** for LTS rule lookahead / -behind
- ▶ **Solution:** construct 2 independent ACPMs

## Information Loss

- ▶ ACPM outputs only **pattern sets**, not input symbols
- ▶ **Problem** for dual-ACPM construction (need both)
- ▶ **Solution:** preserve input in lookbehind ACPM

## Fixed Increment

- ▶ ACPM outputs a pattern set for **each input symbol**
- ▶ **Problem** for LTS **rule-dependent increment**
- ▶ **Solution:** filter out unneeded outputs



# LTS FST: Match Detection

## Left-Context Matching

## (Lookbehind)

$$\begin{aligned} M_L &\cong \mathcal{AC}_{\{\alpha \mid (\alpha[\beta]\gamma \rightarrow \pi) \in R\}} \\ &: \Sigma_w^* \rightarrow (\Sigma_w \times \mathfrak{P}(R))^* \\ &: w \mapsto \bigodot_{i=0}^{|w|} \langle w_i, \{(\alpha[\beta]\gamma \rightarrow \pi) \in R \mid w_{i-|\alpha|..i} = \alpha\} \rangle \end{aligned}$$

## Target & Right-Context Matching

## (Lookahead)

$$\begin{aligned} M_R &\cong \text{reverse}(\mathcal{AC}_{\{(\beta\gamma)^{-1} \mid (\alpha[\beta]\gamma \rightarrow \pi) \in R\}}) \\ &: (\Sigma_w \times \mathfrak{P}(R))^* \rightarrow \mathfrak{P}(R)^* \\ &: \langle w_i, S_i \rangle_I \mapsto \\ & \quad \bigodot_{i \in I} (S_{i-1} \cap \{(\alpha[\beta]\gamma \rightarrow \pi) \in R \mid w_{i..i+|\beta\gamma|} = \beta\gamma\}) \end{aligned}$$

## Match Detection FST

## (Current Match Subset)

$$M_{LR} = M_L \circ M_R : \Sigma_w^* \rightarrow \mathfrak{P}(R)^* : w \mapsto \bigodot_{i=1}^{|w|} R \sim_i w$$



# LTS FST: Output Filter

## Notation

- ▶ Let  $\Sigma_{LR} \subseteq \mathfrak{P}(R)$  be the **output alphabet** of  $M_{LR}$
- ▶ For each  $S \in \Sigma_{LR}$ , let  $(\alpha_S[\beta_S]\gamma_S \rightarrow \pi_S) = \min_{\prec} S$

## Output Filter Construction

- ▶ Define output filter  $M_O : \Sigma_{LR}^* \rightarrow \Sigma_p^*$  as:

$$M_O = \left( \bigcup_{S \in \Sigma_{LR}} \left[ \underbrace{(S : \pi_S)}_{\text{apply}} \underbrace{(\Sigma_{LR} : \varepsilon)^{|\beta_S|-1}}_{\text{increment}} \right] \right)^*$$

## LTS Transducer

- ▶ Then, the **desired LTS FST** can be defined as:

$$M_{LTS} = (M_{LR} \circ M_O) = (M_L \circ M_R \circ M_O) : \Sigma_w^* \rightarrow \Sigma_p^*$$



# LTS FST: Example

**Input:** # s a c h e #



# LTS FST: Example

**Input:** # s a c h e #

---

**M<sub>L</sub>**



# LTS FST: Example

Input: # s a c h e #

---

$M_L$        $\emptyset$   
→



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l} M_L \xrightarrow{\longrightarrow} \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a : , \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \end{array}$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \xrightarrow{\quad} \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}
 \end{array}$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \xrightarrow{\quad} \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}
 \end{array}$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}
 \end{array}$$



# LTS FST: Example

| Input:            | #           | s                                                                                                                                                                                            | a                                                                                                                                                                    | c                                                                                                                                                                                            | h                                                                                                                                                                    | e                                                                                                                                                                    | #                                                                                                                                                                    |
|-------------------|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $M_L \rightarrow$ | $\emptyset$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ |



# LTS FST: Example

| Input:            | #           | s                                                                                                                                                                                            | a                                                                                                                                                                    | c                                                                                                                                                                                            | h                                                                                                                                                                    | e                                                                                                                                                                    | #           |
|-------------------|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|
| $M_L \rightarrow$ | $\emptyset$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\emptyset$ |



# LTS FST: Example

| Input:            | #           | s                                                                                                                                                                                            | a                                                                                                                                                                    | c                                                                                                                                                                                            | h                                                                                                                                                                    | e                                                                                                                                                                    | #           |
|-------------------|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|
| $M_L \rightarrow$ | $\emptyset$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\emptyset$ |

$M_R$

←



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

|                      |             |                                                                                                                                                           |                                                                                                                                 |                                                                                                                                                           |                                                                                                                                 |                                                                                                                                 |             |
|----------------------|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-------------|
| <b>M<sub>L</sub></b> | $\emptyset$ | $[a]ch \rightarrow a$<br>$[a] \rightarrow a ::,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$\#[s]a \rightarrow z,$<br>$[s] \rightarrow s$ | $[a]ch \rightarrow a,$<br>$[a] \rightarrow a ::,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$[s] \rightarrow s$ | $[a]ch \rightarrow a,$<br>$[a] \rightarrow a ::,$<br>$a[ch] \rightarrow x,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$[s] \rightarrow s$ | $[a]ch \rightarrow a,$<br>$[a] \rightarrow a ::,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$[s] \rightarrow s$ | $[a]ch \rightarrow a,$<br>$[a] \rightarrow a ::,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$[s] \rightarrow s$ | $\emptyset$ |
|----------------------|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-------------|

**M<sub>R</sub>**

$\emptyset$

←



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

|                      |             |                                                                                                                                                           |                                                                                                                                 |                                                                                                                                                           |                                                                                                                                 |                                                                                                                                 |             |
|----------------------|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-------------|
| <b>M<sub>L</sub></b> | $\emptyset$ | $[a]ch \rightarrow a$<br>$[a] \rightarrow a ::,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$\#[s]a \rightarrow z,$<br>$[s] \rightarrow s$ | $[a]ch \rightarrow a,$<br>$[a] \rightarrow a ::,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$[s] \rightarrow s$ | $[a]ch \rightarrow a,$<br>$[a] \rightarrow a ::,$<br>$a[ch] \rightarrow x,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$[s] \rightarrow s$ | $[a]ch \rightarrow a,$<br>$[a] \rightarrow a ::,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$[s] \rightarrow s$ | $[a]ch \rightarrow a,$<br>$[a] \rightarrow a ::,$<br>$[c] \rightarrow k,$<br>$[e] \rightarrow \partial,$<br>$[s] \rightarrow s$ | $\emptyset$ |
|----------------------|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-------------|

|                      |                                             |             |
|----------------------|---------------------------------------------|-------------|
| <b>M<sub>R</sub></b> | $\left\{ [e] \rightarrow \partial \right\}$ | $\emptyset$ |
| $\leftarrow$         |                                             |             |



# LTS FST: Example

**Input:** #      s      a      c      h      e      #

---

$$\begin{array}{c}
 \textbf{M}_L \\
 \xrightarrow{\quad} \\
 \emptyset \left\{ \begin{array}{l}
 [a]ch \rightarrow a \\
 [a] \rightarrow a : \\
 [c] \rightarrow k, \\
 [e] \rightarrow \partial, \\
 \#[s]a \rightarrow z, \\
 [s] \rightarrow s
 \end{array} \right\} \left\{ \begin{array}{l}
 [a]ch \rightarrow a, \\
 [a] \rightarrow a : \\
 [c] \rightarrow k, \\
 [e] \rightarrow \partial, \\
 [s] \rightarrow s
 \end{array} \right\} \left\{ \begin{array}{l}
 [a]ch \rightarrow a, \\
 [a] \rightarrow a : \\
 a[ch] \rightarrow x, \\
 [c] \rightarrow k, \\
 [e] \rightarrow \partial, \\
 [s] \rightarrow s
 \end{array} \right\} \left\{ \begin{array}{l}
 [a]ch \rightarrow a, \\
 [a] \rightarrow a : \\
 [c] \rightarrow k, \\
 [e] \rightarrow \partial, \\
 [s] \rightarrow s
 \end{array} \right\} \left\{ \begin{array}{l}
 [a]ch \rightarrow a, \\
 [a] \rightarrow a : \\
 [c] \rightarrow k, \\
 [e] \rightarrow \partial, \\
 [s] \rightarrow s
 \end{array} \right\} \left\{ \begin{array}{l}
 [a]ch \rightarrow a, \\
 [a] \rightarrow a : \\
 [c] \rightarrow k, \\
 [e] \rightarrow \partial, \\
 [s] \rightarrow s
 \end{array} \right\} \emptyset
 \end{array}$$


---

$$\begin{array}{c}
 \textbf{M}_R \\
 \xleftarrow{\quad} \\
 \emptyset \qquad \qquad \qquad \left\{ [e] \rightarrow \partial \right\} \qquad \emptyset
 \end{array}$$



# LTS FST: Example

| Input:            | #           | s                                                                                                                                                                                            | a                                                                                                                                                                     | c                                                                                                                                                                                                                                                                                                                     | h                                                                                                                                                                     | e                                                                                                                                                                     | #           |
|-------------------|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|
| $M_L \rightarrow$ | $\emptyset$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a ::, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a ::, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$<br>$\left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a ::, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a ::, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\}$ | $\emptyset$ |

|                  |                                                                                           |             |                                             |             |
|------------------|-------------------------------------------------------------------------------------------|-------------|---------------------------------------------|-------------|
| $M_R \leftarrow$ | $\left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\}$ | $\emptyset$ | $\left\{ [e] \rightarrow \partial \right\}$ | $\emptyset$ |
|------------------|-------------------------------------------------------------------------------------------|-------------|---------------------------------------------|-------------|



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\mathbf{M_R} \leftarrow \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a : , \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \# [s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a : , \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a : , \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a : , \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a : , \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a : , \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\mathbf{M_R} \leftarrow \left\{ \begin{array}{l} \# [s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a : \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ [e] \rightarrow \partial \right\} \emptyset$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\mathbf{M_R} \leftarrow \emptyset \left\{ \begin{array}{l} \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \textbf{M}_L \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\begin{array}{l}
 \textbf{M}_R \leftarrow \emptyset \left\{ \begin{array}{l} \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset
 \end{array}$$

**M<sub>O</sub>**

→



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\mathbf{M_R} \leftarrow \emptyset \left\{ \begin{array}{l} \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset$$

$$\mathbf{M_O} \xrightarrow{\varepsilon}$$



# LTS FST: Example

**Input:** #      s      a      c      h      e      #

---

$$\begin{array}{l}
 \textbf{M}_L \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$


---

$$\begin{array}{l}
 \textbf{M}_R \leftarrow \emptyset \left\{ \begin{array}{l} \#\![s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset
 \end{array}$$


---

$$\begin{array}{l}
 \textbf{M}_O \rightarrow \varepsilon \quad z
 \end{array}$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\mathbf{M_R} \leftarrow \emptyset \left\{ \begin{array}{l} \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} \textcolor{red}{[a]ch \rightarrow a}, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset$$

$$\mathbf{M_O} \rightarrow \varepsilon \quad z \quad \textcolor{red}{a}$$

# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \textbf{M}_L \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\begin{array}{l}
 \textbf{M}_R \leftarrow \emptyset \left\{ \begin{array}{l} \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset
 \end{array}$$

$$\begin{array}{ccccccccc}
 \textbf{M}_O & & & & & & & & \\
 \rightarrow & \varepsilon & z & a & x & \varepsilon & & &
 \end{array}$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\mathbf{M_R} \leftarrow \emptyset \left\{ \begin{array}{l} \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset$$

$$\mathbf{M_O} \rightarrow \varepsilon \quad z \quad a \quad x \quad \varepsilon \quad \partial$$



# LTS FST: Example

**Input:** #      s      a      c      h      e      #

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$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$


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$$\mathbf{M_R} \leftarrow \emptyset \left\{ \begin{array}{l} \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ [e] \rightarrow \partial \right\} \emptyset$$


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$$\mathbf{M_O} \rightarrow \varepsilon \quad z \quad a \quad x \quad \varepsilon \quad \partial \quad \varepsilon$$



# LTS FST: Example

| Input: | # | s | a | c | h | e | # |
|--------|---|---|---|---|---|---|---|
|--------|---|---|---|---|---|---|---|

$$\begin{array}{l}
 \mathbf{M_L} \rightarrow \emptyset \left\{ \begin{array}{l} [a]ch \rightarrow a \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ a[ch] \rightarrow x, \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \\ [c] \rightarrow k, \\ [e] \rightarrow \partial, \\ [s] \rightarrow s \end{array} \right\} \emptyset
 \end{array}$$

$$\mathbf{M_R} \leftarrow \emptyset \left\{ \begin{array}{l} \#[s]a \rightarrow z, \\ [s] \rightarrow s \end{array} \right\} \left\{ \begin{array}{l} [a]ch \rightarrow a, \\ [a] \rightarrow a :: \end{array} \right\} \left\{ \begin{array}{l} a[ch] \rightarrow x, \\ [c] \rightarrow k \end{array} \right\} \emptyset \left\{ \begin{array}{l} [e] \rightarrow \partial \end{array} \right\} \emptyset$$

$$\mathbf{M_O} \rightarrow \varepsilon \quad z \quad a \quad x \quad \varepsilon \quad \partial \quad \varepsilon$$



# Results

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## The Big Picture

- ▶ Grimm Quotation Evidence Corpus
- ▶ Taxi Corpus Indexing System

## Letter-to-Sound Conversion

- ▶ Festival LTS Rulesets
- ▶ Failed Attempts
  - ... or: “*Why lextools Won’t Cut The Butter*”
- ▶ LTS Transducer Generation

## Results

- ▶ Morphological Coverage

## Demonstration

- ▶ Taxi/Grimm HTTP Query Server



# Results: Performance

## Letter-to-Sound FST

- ▶ Number of LTS Rules : 396
- ▶ Number of States : 1,037
- ▶ Number of Final States : 292
- ▶ Number of Arcs : 131,440
- ▶ Compilation Time : ca. 5 minutes

## Performance

| LTS Method      | Throughput (tok/s) | Relative   |
|-----------------|--------------------|------------|
| festival (TCP)  | 28.53              | -4875.57 % |
| festival (pipe) | 1391.45            | ± 0.00 %   |
| FST (libgfsm)   | 9124.69            | + 555.77 % |



# Results: Grimm Corpus

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## Types (alphabetic, non-foreign)

- ▶ Number of Orthographic Types: 323,561
  - ▶ Morphologically Analyzed : 137,613 (42.53 %)
- ▶ Number of Phonetic Types : 267,933
  - ▶ Morphologically Analyzed : 130,796 (48.82 %)

## Tokens (alphabetic, non-foreign)

- ▶ Number of Tokens : 5,491,978
  - ▶ Orthographic form known: 4,600,619 (83.77 %)
  - ▶ Phonetic form known : 5,011,177 (91.25 %)
- ▶ “Error” Reduction : 410,558 (46.09 %)



# Demonstration

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## The Big Picture

- ▶ Grimm Quotation Evidence Corpus
- ▶ Taxi Corpus Indexing System

## Letter-to-Sound Conversion

- ▶ Festival LTS Rulesets
- ▶ Failed Attempts
  - ... or: “*Why lextools Won’t Cut The Butter*”
- ▶ LTS Transducer Generation

## Results

- ▶ Morphological Coverage

## Demonstration

- ▶ Taxi/Grimm HTTP Server: [localhost:8765](http://localhost:8765)



# Thanks for Listening !

/dəŋ·kə ſfn /!

